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Problem posing with Representation Conversion Model for Learning the Condition of Addition and Subtraction Word Problems

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Abstract: Understanding arithmetic word problems can be said as a structural understanding of the relationship between linguistic and mathematical representations. The goal of this study is to make learners understand the conditions of addition or subtraction word problem. This study proposes an exercise of the conversion among linguistic, mathematical, and graphic representations. Monsakun-TapeBlock is a learning environment to conduct the exercise. The effectiveness of these exercises is suggested through the experimental use in a public elementary school. The learners came to explain the relation between the linguistic and the mathematical representation with the quantity roles and judge valid arithmetic word problems better.

Keywords: Arithmetic Word Problem, meaningful approach, problem posing, graphic representation

1. Introduction

Solving an arithmetic word problem can be said to read sentences, extract the quantity relationship in them, represent it as a mathematical representation, and derive unknown numbers (Mayer, 1992). Many researchers have already investigated solving process of the word problems, and they have divided the process into two sub-processes: (1) comprehension phase and (2) solution phase (Cummins et al., 1988)(Hegarty et al., 1995)(Heffernan and Koedinger, 1998)(Riley et al., 1983). They have also pointed that the comprehension phase plays a major role in the difficulty of the word problems. In this phase, a learner is required to interpret the representation written by words and to create quantitative relationships. Here, several researchers assumed that the product of the comprehension phase is a representation that is connecting "problem (conceptual) representation" and "quantitative representation " (Koedinger and Nathan, 2004)(Nathan et al., 1992)(Reusser, 1996).

There are two general approaches in the comprehension phase: a short-cut approach and a meaningful approach (Hegarty et al., 1995). In the short-cut approach, the problem solver attempts to select the numbers in the problem and critical relational terms (such as "more" and "less" which imply an operation) and make a numeric formula. On the other hand, in the meaningful approach, the problem solver translates the problem statement into a mental model to the situation described in the problem statement. The mental model becomes the basis for the solution phase.

The meaningful approach has two learning methods. One is problem representational technique in problem-solving, and the other is problem posing. Problem representational technique provides learners with schematic diagrams. Schematic diagrams represent problem schema depending on problem types, e.g., change, combine and compare in addition and subtraction word problems (Riley et al., 1983). Schema-based strategy is better than the traditional approaches. Several investigations have confirmed that learning by problem posing in conventional classrooms is a promising activity in learning mathematics (Silver and Cai, 1996; English, 1998). Since learners are usually allowed to pose several kinds of problems, and they can make an extensive range of problems, it is difficult for teachers to complete assessment and feedback for the posed problems in classrooms practically.

The purpose of this study is not to simply teach this conversion as a procedure but to make it empirically understood by tackling various issues in the learning environment. Of course, it is considered that the practice of problem exercises in the current school education is also aimed at understanding empirically after teaching the mechanism. In this study, the mechanism is expressed in a computational form by making it explicit and realizing it as a learning environment on a computer and realizing immediate feedback, and we are trying to create a learning form that has been difficult to achieve by creating more learning opportunities.

This study proposes the use of problem representational technique in problem posing with the learning environment for learning arithmetic word problems by sentence-integration "Monsakun" (Hirashima et al., 2007) (Hirashima et al., 2008) (Hirashima and Kurayama, 2011). In Monsakun, instead of writing sentences freely, learners pose arithmetic word problems as the combination of the provided sentences. The task learners perform in Monsakun is to pose arithmetic word problems satisfying the required condition; numeric formula and the type of story. This task requires learners to make a mental model of a story that can be solved by the provided numeric formula by themselves. This study uses the graphic representation to support this process. The graphic representation can be a guideline to make a mental model. We conducted the experimental use of this learning environment and measured the effectiveness of it.

The research questions in this experimental use are the followings:

RQ1: Do the students improve the understanding of addition and subtraction word problems?
RQ2: Do the students accept the exercise on Monsakun-TapeBlock?
RQ3: Is the exercise appropriate to elementary school students?

In this paper, a study on arithmetic text problems is taken as an example. Solving and creating sum-and-difference arithmetic text problems is defined as a conversion between linguistic and mathematical representation using graphic representation as an intermediate. This paper has the following structure. Section 2 proposes a graphic representation of numerical relationships in the one-step arithmetic word problem. Section 3 shows the learning environment in which learners convert a problem statement to a numeric formula through the graphic representation —section 4 reports a case study of the use of the learning environment in an elementary school. Section 5 concludes this paper.

2. Representation Conversion Model for Arithmetic Word Problems

This study models arithmetic story and word problems as the relationship between the linguistic, graphical, and mathematical representations shown in Fig. 1, based on the Triplet-structure model (Hirashima et al., 2014). The linguistic representation of an arithmetic story is the problem statement, for example, “Three flowers were in bloom. Two flowers bloomed. Five flowers are in bloom.” We can make a word problem “Three flowers were in bloom. Two flowers bloomed. How many flowers are in bloom?” by change a number to unknown.

In the model, the linguistic representation is the composition of three simple sentences expressing a quantity relationship to clarify the roles of the quantities in the statement. Each simple sentence has a quantity (in this figure, the number of flowers), and each quantity has a role depending on the meaning of the statement. In the case of Figure 1, the quantity of three flowers has the role of a *start* quantity, the quantity of two flowers has the role of a *change* quantity, and the quantity of five flowers has the role of *end* quantity in the story.

Based on the roles of quantities in this linguistic representation, we can identify the relationships among the quantities in which the number of flowers increased from 3 to 5 by adding 2 and make the numerical formula "3 + 2 = 5" a mathematical representation. In addition to that, we can also make the formulas "5 - 3 = 2" and "5 - 2 = 3". This kind of process is a part of the comprehension process of an arithmetic stories and word problems. Problem-solving is to convert a mathematical representation into the linguistic one, and problem posing is to convert a linguistic representation into the mathematical one. The relation among the quantities can be the bridge between linguistic and mathematical representations, and identifying the relation can be an important task in both problem solving and posing.

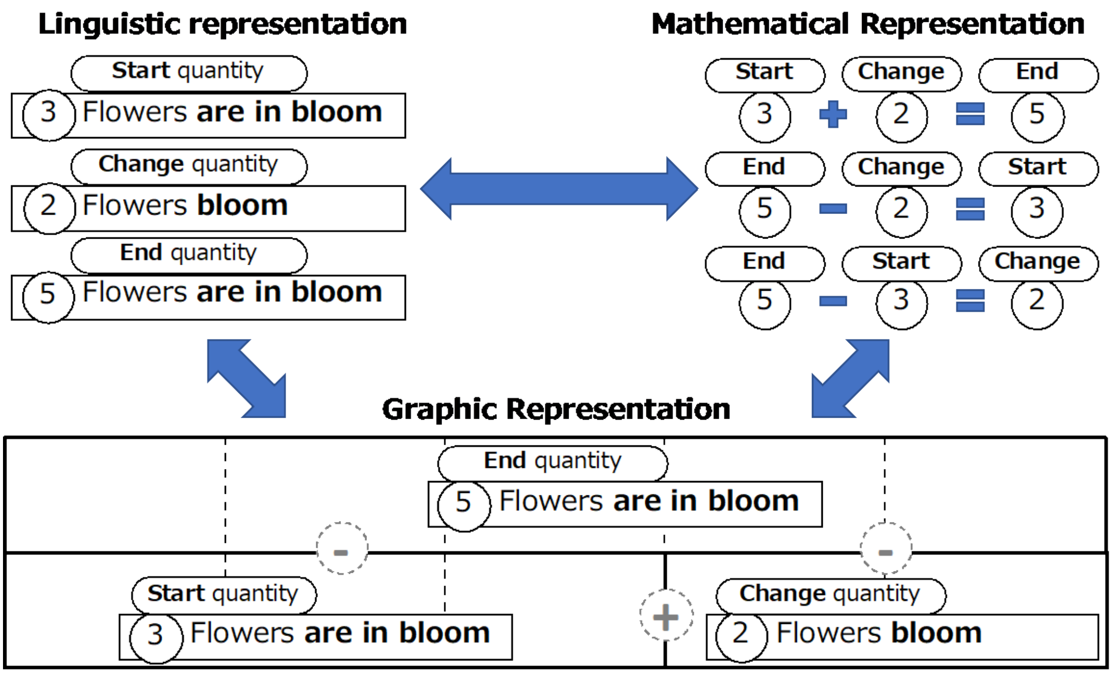

Figure 1. The relationship between the linguistic, graphical, and mathematical representations.

This study proposes "Tape-block" as the graphic representation of the relation among the quantities. The bottom of Fig. 1 shows the proposed graphic representation based on part-part-whole schema and drawing (Resnick, 1983)(Willis and Fuson, 1988). This representation itself describes the relation among three quantities, in which the top quantity is the sum of the two bottom quantities. This also describes the difference between the top quantity and a bottom quantity in the other bottom quantity. This means that a numerical relation in this type can be converted to three types of equality: one addition and two subtractions shown at the top-left in Fig. 1. The correspondence between the graphical and the mathematical representation is by the magnitude of quantities. The largest number in the mathematical representation must be located at the top of the graphic representation. On the other hand, the correspondence between the graphical and the linguistic representation is by the meaning of the quantities. The resultant quantity in the linguistic representation must be located at the top of the graphic representation. These correspondences can explain the correspondence between the linguistic and the mathematical representation.

Figure 2 shows the classification of arithmetic stories and the comparison between the schematic drawing and Tape-block diagram. There are four types of arithmetic word problems: Put-together (combine), change-get-more, change-get-less, and compare. Schematic drawings define four different drawings depending on the type of arithmetic word problems. Tape-block diagram uses the same diagram for all the type of arithmetic word problems. This diagram also implies the algebraic operators between the quantities. For example, in Change-get-more, the addition operator between *Start* and *Change* means the equation: *Start* quantity + *Change* quantity = *End* quantity. Based on the diagram, learners can consider valid equations among three quantities without algebraic manipulations, such as transposition of terms, which elementary school students have not learned.

3. Monsakun-TapeBlock: An Environment for The Exercise of Conversion from Linguistic to Mathematical Representation with The Graphic Representation

Based on the model of the conversion among linguistic, mathematical, and graphic representation with a Tape-block diagram, we developed a learning environment named Monsakun-TapeBlock. This has the exercises of all the types of conversion of linguistic, mathematical, and graphic representation and quantity role assignment in each representation. Here, due to the limitation of the space, we explain the problem-posing process in Monsakun-TapeBlock as an example. Fig.3 to Fig.6 shows the screenshot of the conversion from mathematical representation to linguistic one through graphic one and quantity role assignment in the process.

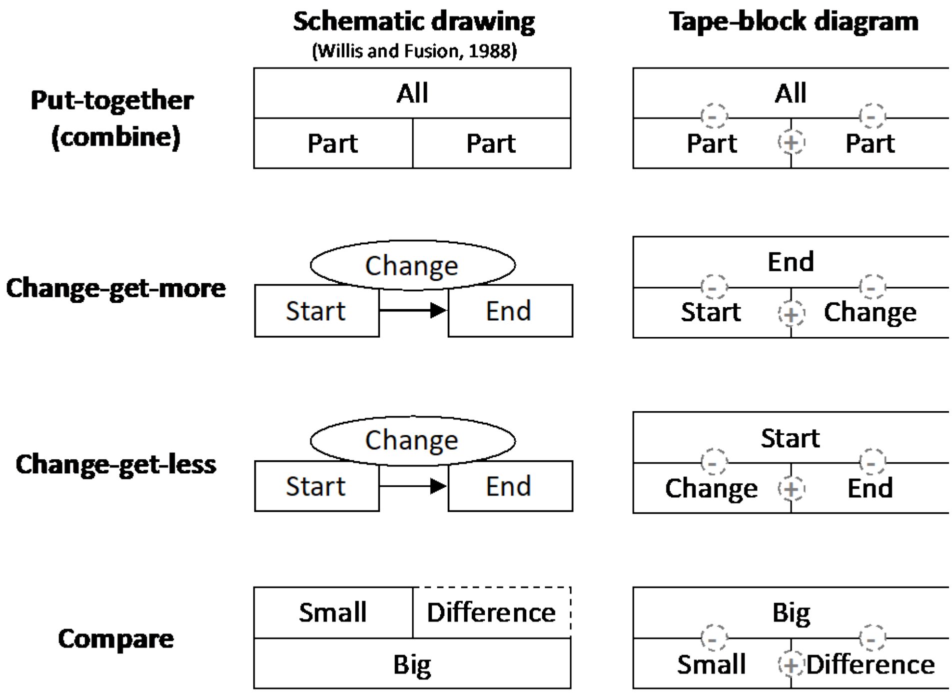

Figure 2. The classification of arithmetic stories.

Figure 3 shows the conversion from mathematical representation to graphic one. Here, a learner put number cards "2", "4" and "6" extracted from the equation "4 + 2 = 6" into the Tape-block diagram. This is just confirmation of the rule of the Tape-block diagram: the largest number must be the top.

Figure 4 shows the quantity role assignment to numbers in graphic representation. Learners are required to assign quantity roles to numbers in graphic representation. Here, "all," "part" and "part" role quantity, that are roles in a put-together story, are provided to learners, and they are required to consider the assignment based on the rule of the story; "all" quantity role must be the top in a put-together story.

Figure 5 shows the conversion from linguistic representation to graphic one by reference to numbers in graphic representation. The required task here is to decide two things; the position of numbers and quantity roles in the sentences. This is not only just the matching of numbers but also matching quantity roles between sentence and graphic representation. Low understanding students tend to only match numbers. The purpose of this task is to facilitate them to also consider the quantity roles in graphic representation combined with concrete sentences.

Figure 6 shows the quantity role assignment to sentences in graphic representation. In this procedure, this task is a review of the previous task. The purpose of this task is to ask them to explain what they have made in the previous task. Learners are required to assign quantity roles to sentences in the graphic representation. Through this task, we expect learners to reflect on why the posed problem is correct. In Normal Monsakun, the feedback is correct or not, and if the answer is incorrect, feedback about the reason for the error is provided. When the answer is correct, some learners just look at the feedback and do not confirm their answer. To avoid this, in Monsakun-TapeBlock, this task is included.

This procedure is just an example. There can be other procedures to conduct the conversion of representations and quantity role assignment. For example, not like the above procedure, we can compose the procedure where learners conduct the quantity role assignment to sentences in linguistic representation and then the conversion from linguistic representation to graphic one. In this case, the quantity role assignment is not a review but guidelines to consider the position of sentences in graphic representation. Like this, a different combination of conversion and role assignment can make different task sequences for different learning goals.

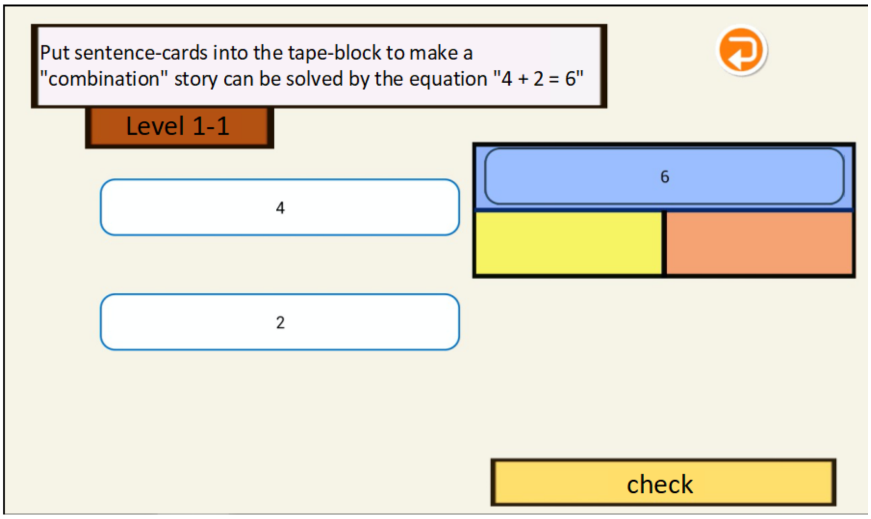

Figure 3. Conversion from mathematical representation to graphic one.

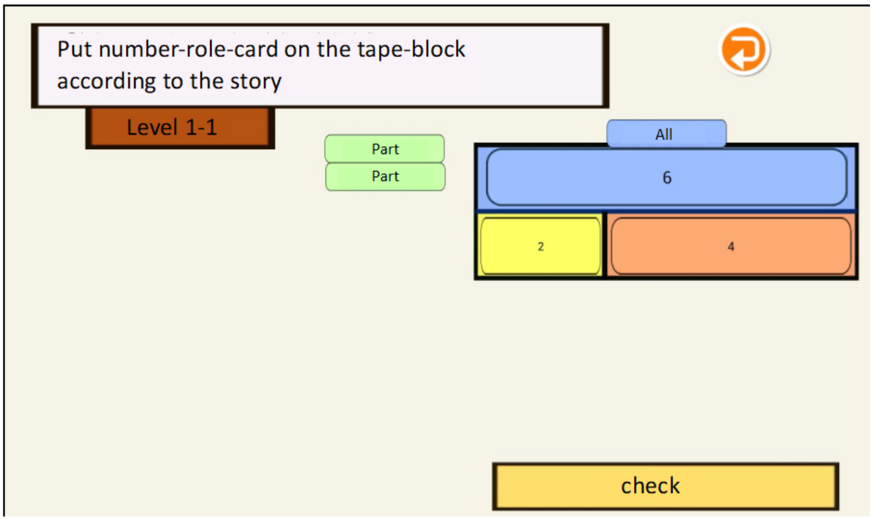

Figure 4. Quantity role assignment to numbers in graphic representation.

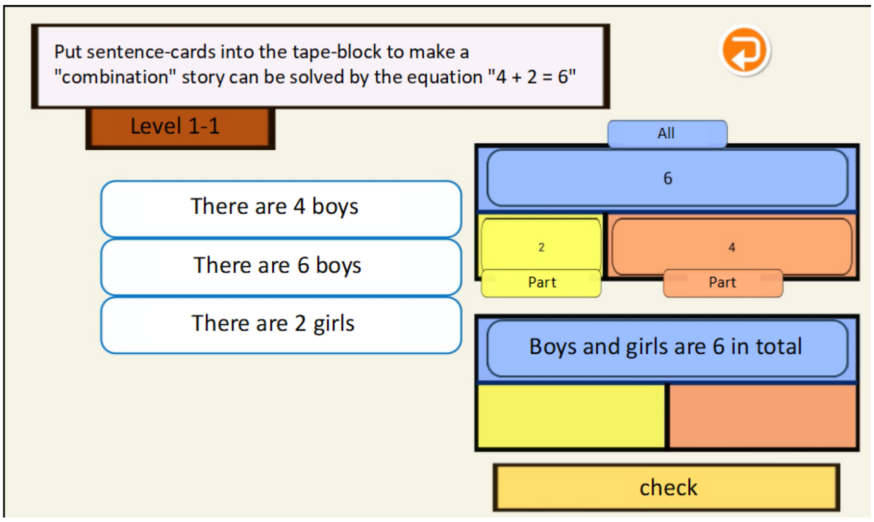

Figure 5. Conversion from graphic representation to a linguistic one.

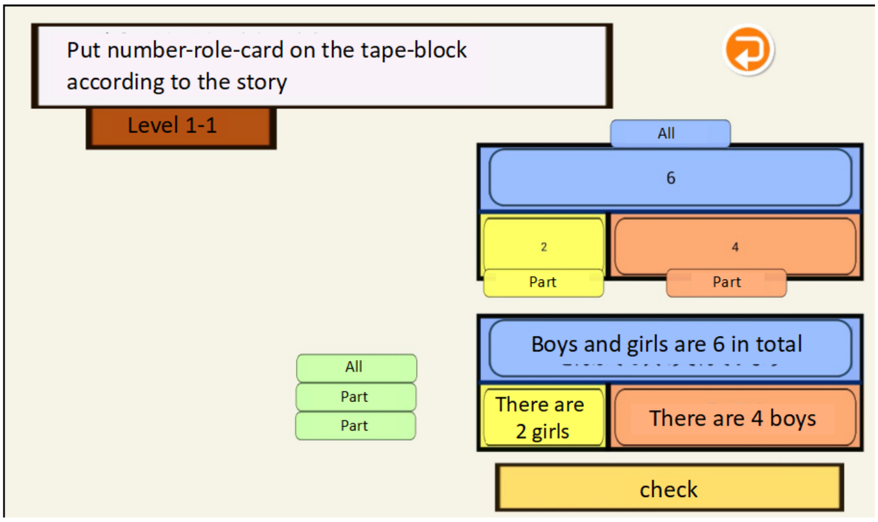

Figure 6. Quantity role assignment in graphic representation with quantity propositions.

4. Experimental Use in An Elementary School

4.1 Purpose and Procedure

To answer the research questions mentioned above, we conducted pre/post-tests and questionnaires in addition to the exercise on Monsakun-TapeBlock. Eighty-eight students in third-grade public elementary school students (8-9 years old) use Monsakun-TapeBlock in two units of lessons. The procedure is the following:

1. a quick review of addition and subtraction word problems (5 min)
2. pre-test (15 min)
3. exercise on Monsakun-TapeBlock (50 min)
4. post-test (15 min)
5. questionnaire (5 min)

To answer RQ1, we develop the structure comprehension test for the pre/post-tests named "priming test." The priming test asks learners whether the shown story is valid or not. For example, "Jon has 3 apples. Jon and Bill have 8 apples altogether. Bill has 5 apples." is valid. On the other hand, "Jon has 3 apples. Jon and Bill have 5 apples altogether. Bill has 8 apples." and "Jon has 3 apples. Jon and Bill have 8 apples altogether. Bill has 5 oranges." are not valid. An item shows firstly two sentences and then shows the last sentence. This task requires the understanding of the conditions of addition or subtraction word problem. During the first part, if learners can predict the last sentence, the learners can quickly answer the validity of the story. If they start to think after the display of the last sentence, they take much time to answer. Figure 7 and 8 show the screenshots of it. The progress bar shows the time to display the last sentence. When the bar reaches the right end, the last sentence is displayed shown in Figure 8. This test has 13 items, including four kinds of story: put-together, change-get-more, change-get-less, and compare.

To answer RQ2, we carried out the questionnaire includes the following:

1. Do you like to study mathematics?
2. Did you use Monsakun-TapeBlock easily?
3. Did you enjoy using Monsakun-TapeBlock?
4. Do you think posing problems in Monsakun-TapeBlock is good for studying mathematics?
5. Do you think quantity role assignment in Monsakun-TapeBlock is good for studying mathematics?
6. Is it easy for you to assign quantity roles in Monsakun-TapeBlock?

To answer RQ3, we analyzed the relation between pre-test score and the performance in the exercise on Monsakun-TapeBlock, and log data in the exercise on Monsakun-TapeBlock. If the exercise requires the understanding of the conditions of addition or subtraction word problems, learners getting a high score in the pre-test can show a good performance in the exercise. In addition to that, if the exercise is effective, learners' performance is improved through the exercise. We focus on the first level in Monsakun-TapeBlock. All the students participated in this experimental use completed all the exercise at this level. This level has 12 exercises in total, and they are composed of three sets. Each set has four exercises about the four types of addition and subtraction word problems: put-together, change-get-more, change-get-less, and compare. Each exercise has four steps of tasks to pose a problem as follows.

Step1: Conversion from mathematical representation to graphic representation: putting the only number to Tape-block (Fig. 3),
Step2: Association of Quantity role to numbers in Tape-block,
Step3: Conversion from graphic representation to linguistic representation: putting sentences to Tape-block (Fig. 4),
Step4: Association of Quantity role to sentences in Tape-block (Fig. 6).

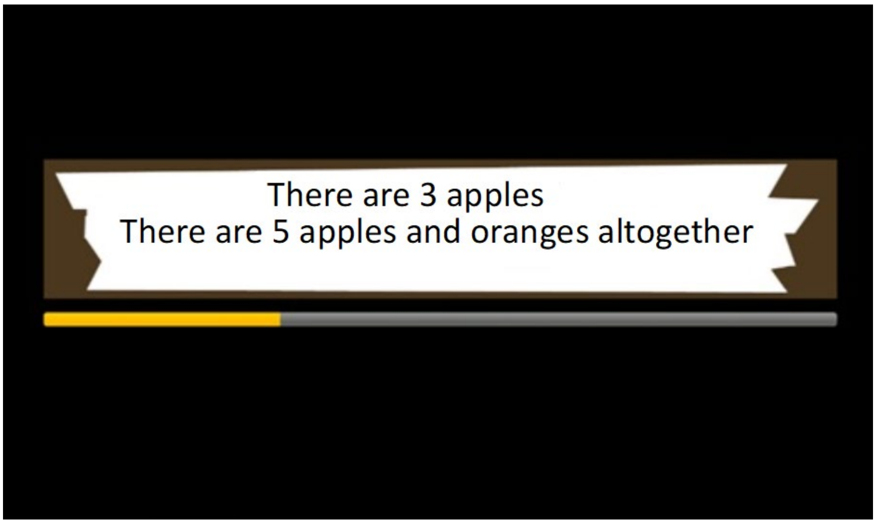

Figure 7. A screenshot of the first step in the Priming test.

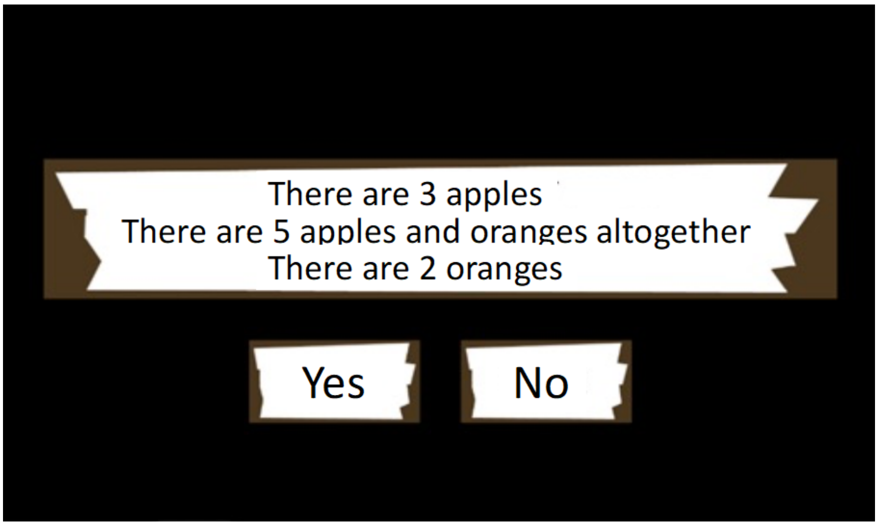

Figure 8. A screenshot of the last step in the Priming test.

4.2 Results and Consideration

Table 1 shows the result of the pre/post-test. We analyze the difference of score and time between pre and post-test with the Wilcoxon signed-rank test. There is not a significant difference in the score. On the other hand, there is a significant difference in time. This means that they keep improving the understanding of addition and subtraction word problems in terms of the speed of thinking. Unfortunately, they did not get worse and improve in the correctness. We answer partially yes to RQ1: "Do the students improve the understanding of addition and subtraction word problems?" is yes.

Table 1. *The Result of Pre/Post-Test.*

	Pre mean (sd)	Post mean (sd)	p-value
Score (full score is 13)	10.33 (2.32)	10.31 (2.16)	0.9468
Average time per item (sec)	5.77 (3.38)	4.62 (2.65)	0.0007

Figure 9 shows the result of the questionnaire. From the question 1-3 most of the student like to study mathematics and enjoy the exercise on Monsakun-TapeBlock with easy use. From the question 4-5, most of them enjoyed the exercise and felt the effectiveness of it. From the last question, the exercise is not always easy for the students. From these results, the students accept the exercise on Monsakun-TapeBlock as enjoyable and useful. We answer yes to the RQ2: "Do the students accept the exercise on Monsakun-TapeBlock?"

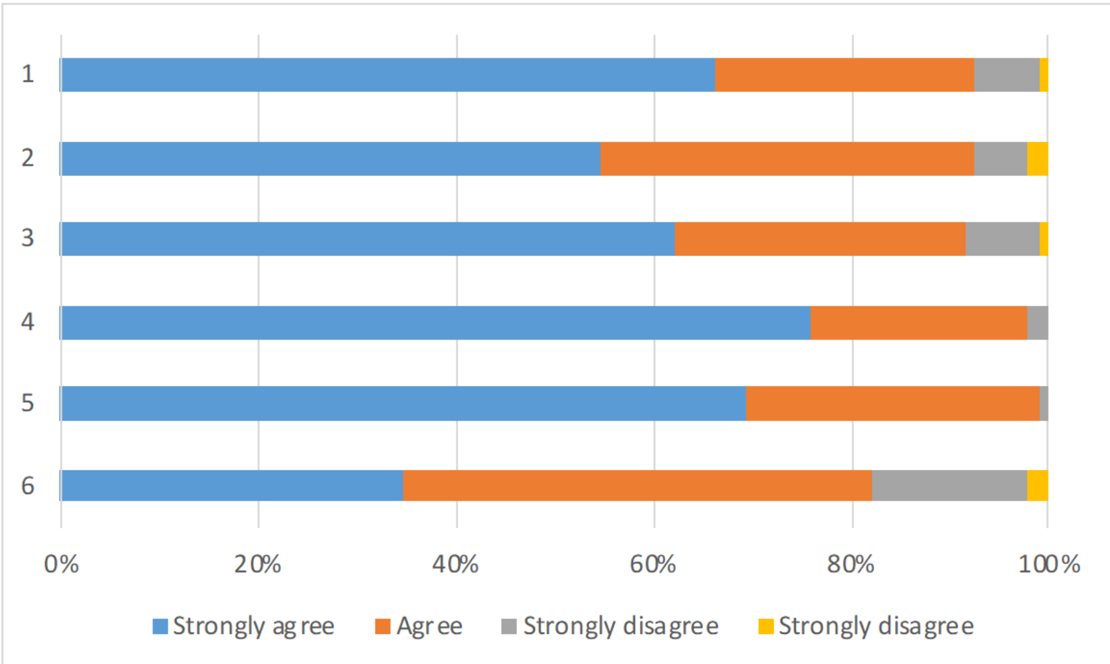

Figure 9. The result of the questionnaire.

To investigate what happened in the exercise, we analyze the log data recorded in the exercise. The log data includes all the answers of students in the exercise. Firstly, we analyzed whether the exercise is appropriate to the students and then their change in the exercise. The target data is the first level in Monsakun-TapeBlock. All the students can complete this level. At this level, students pose problems from mathematical representation to linguistic one through the graphic one discussed in Section 3.

Table 2 shows the difference in mean of check-in the exercise between the lower and higher groups. In Monsakun-TapeBlock, learners repeat to answer until they get to the correct answer. In this analysis, we categorized the learners by the mean of pre-test score and compare the mean of check-in Monsakun-TapeBlock by Wilcoxon rank-sum test. The result shows that lower group students need significantly much more check times to get the correct answers than the higher group students. This means the exercise in Monsakun-TapeBlock requires the understanding of the conditions of addition or subtraction word problem can be measured with the Priming test.

Table 2. *The Result of Post-Test Devided into Higher/Lower Groups.*

	Lower group	Higher group	p-value
mean of check (times)	1.55 (0.52)	1.25 (0.32)	0.0000

Tables 3 and 4 show the detailed analysis results. Tasks are the representation conversion or quantity role assignment. Task 1 is the conversion from the mathematical representation to the graphic one shown in Fig. 3. Task 2 is the quantity role assignment in graphic representation with quantities shown in Fig. 4. Task 3 is the conversion from graphic representation to the linguistic one shown in Fig. 5. Task 4 is the quantity role assignment in graphic representation with quantity propositions shown in Fig. 6. At the first level in Monsakun-TapeBlock, learners conduct these four tasks for problem posing. A set of problem posing consists of four types of stories: get-together, change-get-more, change-get-less, and compare. Learners conduct three sets of problem posing.

The comparison between the lower and higher group learners shows there is more change in the lower group than the higher group. In the lower group, the number of checks in Task 1 and Task 4 decreased. On the other hand, In the lower group, only the number of checks in Task 1 decreased. This shows, even in Task 1, although both groups of learners made mistakes, they can improve the performance, and the lower group students improved the performance in Task 4. Though students in both groups have difficulty in Task 3, that is, the conversion from graphic representation to linguistic one, the higher group students can identify the quantity roles from the beginning, and the lower group students improved the performance through the exercise. This means they come to explain the story they posed in the exercise. Therefore, we answer yes to the RQ3: "Is the exercise appropriate to elementary school students?" from the results shown in Table 2, Table 3 and Table 4.

Table 3. *Performance of Lower Group Learners in the Pre-Test.*

		Tasks (mean of check per item)							
		1	p-value	2	p-value	3	p-value	4	p-value
Set	1	1.32		1.64		1.79	0.040	1.53	0.048
	2	1.35		1.43		2.33		1.48	
	3	1.16	0.079	1.57		1.77	0.018	1.16	0.025

Table 4. *Performance of Higher Group Learners in the Pre-Test.*

		Tasks (mean of check per item)							
		1	p-value	2	p-value	3	p-value	4	p-value
Set	1	1.23	0.005	1.30		1.42		1.11	
	2	1.08		1.22		1.48		1.11	
	3	1.05	0.005	1.24		1.58		1.13	

5. Conclusion

Understanding arithmetic word problems can be said as a structural understanding of the relationship between linguistic and mathematical representations. To facilitate learners to build this understanding, this study design and developed a learning environment in which graphic representation bridges the linguistic and the mathematical representation with the quantity roles. The goal of this study is to make learners understand the conditions of addition or subtraction word problem. For the goal, we propose the task of the conversion among linguistic, mathematical, and graphic representations and design and development a learning environment where learners can conduct the exercise of the conversions. The effectiveness of these exercises is suggested through the experimental use in a public elementary school. The learners came to explain the relation between the linguistic and the mathematical representation with the quantity roles and judge valid arithmetic word problems better than before they use this learning environment.

Future tasks will be to verify the difference in learning effect depending on whether or not there is a quantity role matching exercise. In addition to the effects of learning to appear, it is necessary to confirm whether the learner's thought intended in this study appears as the cause.

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