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Learning the Condition of Addition and Subtraction Word Problems by Problem-Posing Based on Representation Conversion Model

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Abstract: Understanding arithmetic word problems can be said as a structural understanding of the relationship between linguistic and mathematical representations. The goal of this study is to make learners understand the conditions of addition or subtraction word problem. This study proposes an exercise of the conversion among linguistic, mathematical, and graphic representations. Monsakun-TapeBlock is a learning environment to conduct the exercise. The effectiveness of these exercises is suggested through the experimental use in a public elementary school. The learners came to explain the relation between the linguistic and the mathematical representation with the quantity roles and judge valid arithmetic word problems better.

Keywords: Arithmetic Word Problem, meaningful approach, Problem-posing, Graphic representation

1. Introduction

Solving an arithmetic word problem can be said to read sentences, extract the quantity relationship in them, represent it as a mathematical representation, and derive unknown numbers (Mayer, 1992). Many researchers have already investigated solving process of the word problems, and they have divided the process into two sub-processes: (1) comprehension phase and (2) solution phase (Cummins et al., 1988; Hegarty et al., 1995; Heffernan and Koedinger, 1998; Pólya, 1945; Riley et al., 1983). They have also pointed that the comprehension phase plays a major role in the difficulty of the word problems. In this phase, a learner is required to interpret the representation written by words and to create quantitative relationships. Here, several researchers assumed that the product of the comprehension phase is a representation that is connecting "problem (conceptual) representation" and "quantitative representation " (Koedinger & Nathan, 2004; Nathan et al., 1992; Reusser, 1996).

There are two general approaches in the comprehension phase: a short-cut approach and a meaningful approach (Hegarty et al., 1995). In the short-cut approach, the problem solver attempts to select the numbers in the problem and critical relational terms (such as "more" and "less" which imply an operation) and make a numeric formula. On the other hand, in the meaningful approach, the problem solver translates the problem statement into a mental model to the situation described in the problem statement. The mental model becomes the basis for the solution phase.

The meaningful approach has two learning methods. One is problem representational technique in problem-solving, and the other is problem-posing. Problem representational technique provides learners with schematic diagrams. Schematic diagrams represent problem schema depending on problem types, e.g., change, combine and compare in addition and subtraction word problems (Riley et al., 1983). Schema-based strategy is better than the traditional approaches. Several investigations have confirmed that learning by problem-posing in conventional classrooms is a promising activity in learning mathematics (Silver & Cai, 1996; English, 1998). Since learners are usually allowed to pose several kinds of problems, and they can make an extensive range of problems, it is difficult for teachers to complete assessment and feedback for the posed problems in classrooms practically.

This study proposes the addition of problem representational technique in problem-posing with the learning environment for learning arithmetic word problems by sentence-integration "Monsakun" (Hirashima et al., 2007; Hirashima et al., 2008; Hirashima & Kurayama, 2011) as a reflection after problem-posing. In Monsakun, instead of writing sentences freely, learners pose arithmetic word problems as the combination of the provided sentences. The task learners perform in Monsakun is to pose arithmetic word problems satisfying the required condition; numeric formula and the type of story. This task requires learners to make a mental model of a story that can be solved by the provided numeric formula by themselves. This study uses the graphic representation to support this process in which learners can check the validity of the posed problems.

This paper has the following structure. Section 2 proposes a graphic representation of numerical relationships in the one-step arithmetic word problem and shows the learning environment in which learners convert a problem statement to a numeric formula through the graphic representation. Section 3 reports a case study of the use of the learning environment in an elementary school. Section 4 concludes this paper.

2. Representation Conversion Model for Arithmetic Word Problems

This study models arithmetic word problems as the relationship between the linguistic, graphical, and mathematical representations shown in Fig. 1, based on the Triplet-structure model (Hirashima et al., 2014). The linguistic representation of an arithmetic word problem is the problem statement, for example, “Three flowers were in bloom. Two flowers bloomed. Five flowers are in bloom.” In the model, the linguistic representation is the composition of three simple sentences expressing a quantity relationship to clarify the roles of the quantities in the statement. Each simple sentence has a quantity (in this figure, the number of flowers), and each quantity has a role depending on the meaning of the statement. In the case of Figure 1, the quantity of three flowers has the role of a *start* quantity, the quantity of two flowers has the role of a *change* quantity, and the quantity of five flowers has the role of *end* quantity in the story.

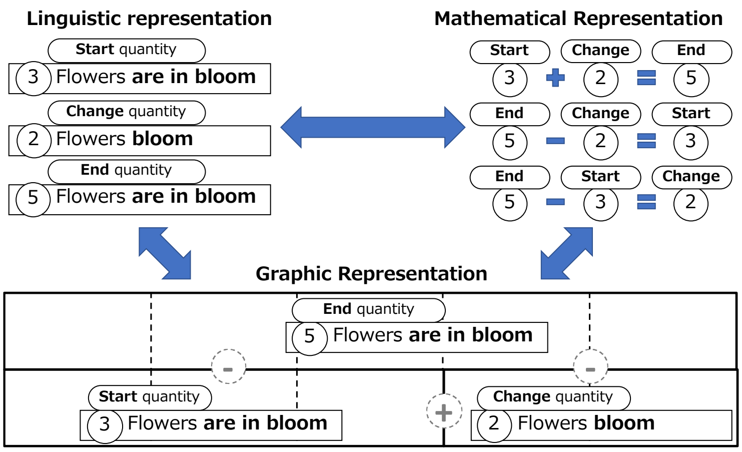

Figure 1. The Relationship between the Linguistic, Graphical, and Mathematical Representations.

This study proposes “Tape-block” as the graphic representation of the relation among the quantities. The bottom of Fig. 1 shows the proposed graphic representation based on part-part-whole schema and drawing (Resnick, 1983; Willis & Fuson, 1988). This representation itself describes the relation among three quantities, in which the top quantity is the sum of the two bottom quantities. This also describes the difference between the top quantity and a bottom quantity in the other bottom quantity. This means that a numerical relation in this type can be converted to three types of equality: one addition and two subtractions shown at the top-left in Fig. 1. The correspondence between the graphical and the mathematical representation is by the magnitude of quantities. The largest number in the mathematical representation must be located at the top of the graphic representation. On the other hand, the correspondence between the graphical and the linguistic representation is by the meaning of the quantities. The resultant quantity in the linguistic representation must be located at the top of the graphic representation. These correspondences can explain the correspondence between the linguistic and the mathematical representation.

Figure 2 shows the classification of arithmetic word problems and the comparison between the schematic drawing and Tape-block diagram. There are four types of arithmetic word problems: Put-together (combine), change-get-more, change-get-less, and compare. Schematic drawings define four different drawings depending on the type of arithmetic word problems. Tape-block diagram uses the same diagram for all the type of arithmetic word problems. This diagram also implies the algebraic operators between the quantities. For example, in Change-get-more, the addition operator between *Start* and *Change* means the equation: *Start* quantity + *Change* quantity = *End* quantity. Based on the diagram, learners can consider valid equations among three quantities without algebraic manipulations, such as transposition of terms, which elementary school students have not learned.

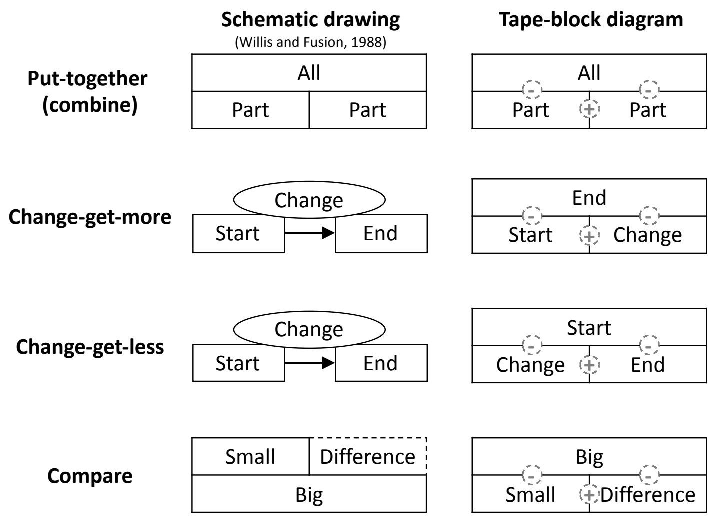

Figure 2. The Classification of Arithmetic Word Problems.

Based on the model of the conversion among linguistic, mathematical, and graphic representation with a Tape-block diagram, we developed a learning environment named Monsakun-TapeBlock. This has the exercises of all the types of conversion of linguistic, mathematical, and graphic representation and quantity role assignment in each representation. Here, due to the limitation of the space, we explain the problem-posing process in Monsakun-TapeBlock as an example. Fig.3 to Fig.6 shows the screenshot of the conversion from mathematical representation to linguistic one through graphic one and quantity role assignment in the process.

3. Experimental Use in an Elementary School

We conducted the experimental use of this learning environment and measured the effectiveness of it.

The research questions in this experimental use are the followings:

RQ1: Do the students improve the understanding of addition and subtraction word problems?

RQ2: Do the students accept the exercise on Monsakun-TapeBlock?

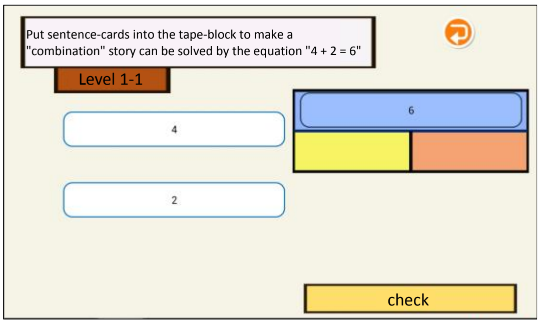

Figure 2. Conversion from Mat Hematical Representation to Graphic One.

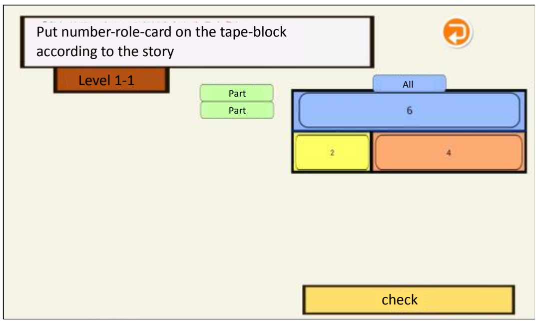

Figure 3. Quantity Role Assignment to Numbers in Graphic Representation.

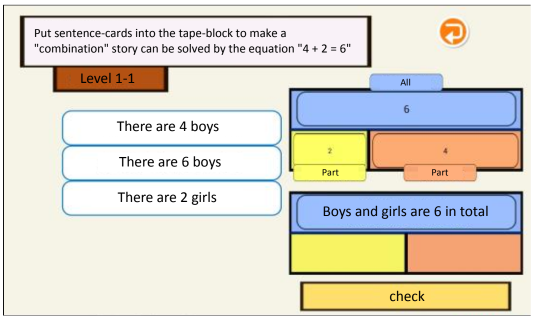

Figure 4. Conversion from Graphic Representation to a Linguistic One.

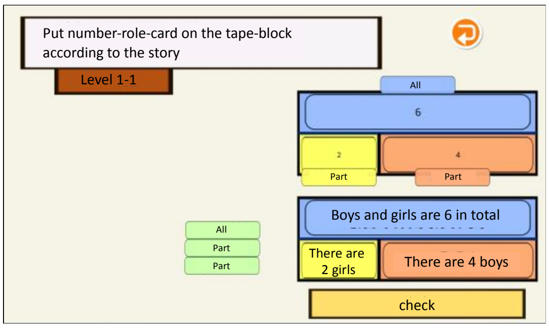

Figure 5. Quantity Role Assignment in Graphic Representation with Quantity Propositions.

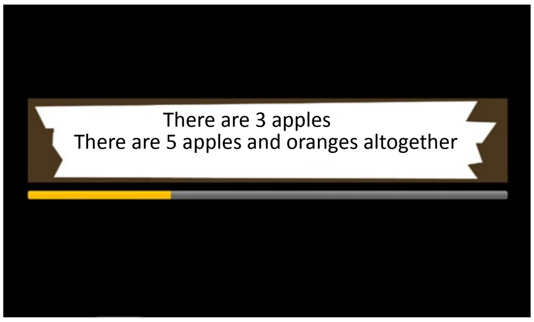

Figure 6. A Screenshot of the First Step in the Priming Test.

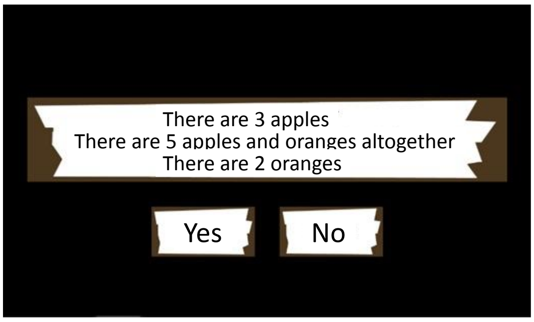

Figure 7. A Screenshot of the Last Step in the Priming Test.

To answer these research questions, we conducted pre/post-tests and questionnaires in addition to the exercise on Monsakun-TapeBlock. Eighty-eight students in third-grade public elementary school students use Monsakun-TapeBlock in two units of lessons. The procedure is the following:

1. a quick review of addition and subtraction word problems (5 min).
2. pre-test (15 min).
3. exercise on Monsakun-TapeBlock (50 min).
   firstly problem-posing, secondary the conversion from linguistic to graphical, and lastly the conversion from graphical to mathematical representations to check the validity of the posed problems.
4. post-test (15 min).
5. questionnaire (5 min).

To answer RQ1, we develop the structure comprehension test for the pre/post-tests named "priming test." The priming test asks learners whether the shown story is valid or not. For example, "Jon has 3 apples. Jon and Bill have 8 apples altogether. Bill has 5 apples." is valid.

On the other hand, “Jon has 3 apples. Jon and Bill have 5 apples altogether. Bill has 8 apples.” and “Jon has 3 apples. Jon and Bill have 8 apples altogether. Bill has 5 oranges.” are not valid. An item shows firstly two sentences and then shows the last sentence. This task requires the understanding of the conditions of addition or subtraction word problem. During the first part, if learners can predict the last sentence, the learners can quickly answer the validity of the story. If they start to think after the display of the last sentence, they take much time to answer. **Error! Reference source not found.** and 8 show the screenshots of it. The progress bar shows the time to display the last sentence. When the bar reaches the right end, the last sentence is displayed shown in **Error! Reference source not found.**. This test has 13 items, including four kinds of story: put-together, change-get-more, change-get-less, and compare.

We answer partially yes to RQ1: “Do the students improve the understanding of addition and subtraction word problems?” is yes. Table 1 shows the result of the pre/post-test. We analyze the difference of score and time between pre and post-test with the Wilcoxon signed-rank test. There is not a significant difference in the score. On the other hand, there is a significant difference in time. This means that they keep improving the understanding of addition and subtraction word problems in terms of the speed of thinking. Unfortunately, they did not get worse and improve in the correctness.

Table 1. *The Result of Pre/Post-Test*

	Pre mean (sd)	Post mean (sd)	p-value
Score (full score is 13)	10.33 (2.32)	10.31 (2.16)	0.9468
Average time per item (sec)	5.77 (3.38)	4.62 (2.65)	0.0007

To answer RQ2, we carried out the questionnaire includes the following:

1. Do you like to study mathematics?
2. Did you use Monsakun-TapeBlock easily?
3. Did you enjoy using Monsakun-TapeBlock?
4. Do you think posing problems in Monsakun-TapeBlock is good for studying mathematics?
5. Do you think quantity role assignment in Monsakun-TapeBlock is good for studying mathematics?
6. Is it easy for you to assign quantity roles in Monsakun-TapeBlock?

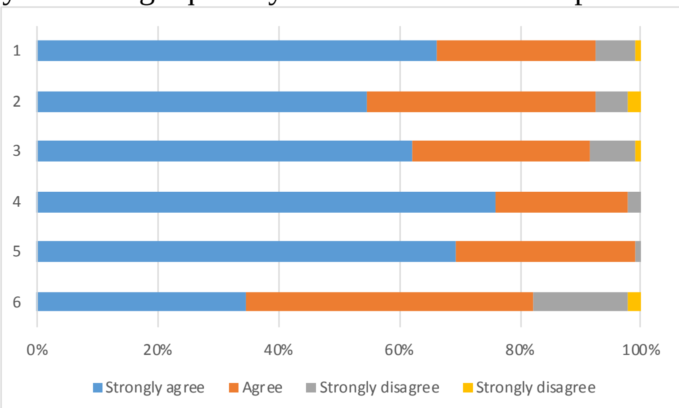

Figure 8. The Result of the Questionnaire

We answer yes to the RQ2: “Do the students accept the exercise on Monsakun-TapeBlock?” ***Figure 8*** shows the result of the questionnaire. From the question 1-3 most of the student like to study mathematics and enjoy the exercise on Monsakun-TapeBlock with easy use. From the question 4-5, most of them enjoyed the exercise and felt the effectiveness of it. From the last question, the exercise is not always easy for the students. From these results, the students accept the exercise on Monsakun-TapeBlock as enjoyable and useful.

4. Conclusion

Understanding arithmetic word problems can be said as a structural understanding of the relationship between linguistic and mathematical representations. To facilitate learners to build this understanding, this study design and developed a learning environment in which graphic representation bridges the linguistic and the mathematical representation with the quantity roles. The goal of this study is to make learners understand the conditions of addition or subtraction word problem with problem-posing and reflections on the posed problems. For the goal, we propose the task of the conversion among linguistic, mathematical, and graphic representations after problem-posing and design and development a learning environment where learners can conduct the exercise of the conversions. The effectiveness of these exercises is suggested through the experimental use in a public elementary school.

Future tasks will be to verify the difference in learning effect depending on whether or not there is a quantity role matching exercise. In addition to the effects of learning to appear, it is necessary to confirm whether the learner's thought intended in this study appears as the cause.

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